RECTILINEAR MOTION

MECHANICS

Mechanics is the branch of physics which deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc. Mechanics is classified under two streams namely ***Kinematics*** and ***Dynamics***.

Mechanics

Kinematics

The word kinematics means 'science of motion'.branch of mechanics which deals with study of motion without going into the cause of motion, i.e. force, torque etc.

Dynamics (or **Kinetics)**

It is branch of mechanics which is concerned about the causes (i.e. the force , torque) that cause motion of bodies.

1. MOTION AND REST

Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to a observer, if its position changes with respect to that observer. It may happen by both ways either observer moves or object moves.

2. RECTILINEAR MOTION

Rectilinear motion is motion, along a straight line or in one dimension. It deals with the kinematics of a particle in one dimension.

2.1 Position

The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the question – *" where is the particle at a particular moment of time ? "*

2.2 Displacement

The change in the position of a moving object is known as displacement. It is the vector joining the initial position of the particle to its final position during an interval of time.

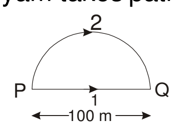
2.3 Distance

The length of the actual path travelled by a particle during a given time interval is called as distance. The distance travelled is a scalar quantity which is quite different from displacement. In general, the distance travelled between two points may not be equal to the magnitude of the displacement between the same points.

Example 1. Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).

(a) Find the distance travelled by Ram and Shyam?

(b) Find the displacement of Ram and Shyam?



Sol. (a) Distance travelled by Ram = 100 m

Distance travelled by Shyam = π(50 m) = 50π m

(b) Displacement of Ram = 100 m

Displacement of Shyam = 100 m

2.4 Average Velocity (in an interval) :

The average velocity of a moving particle over a certain time interval is defined as the displacement divided by the lapsed time.

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$$\text{Average Velocity} = \frac{\text{displacement}}{\text{time interval}}$$

for straight line motion, along x–axis, we have

$$v_{av} = \overline{v} = \langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The average velocity is a vector in the direction of displacement. For motion in a straight line, directional aspect of a vector can be taken care of by +ve and -ve sign of the quantity.

2.5 Average Speed (in an interval)

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place. It helps in describing the motion along the actual path.

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

NOTE:

(a) Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.

(b) If the motion of a particle is along a straight line and in same direction then, average velocity = average speed.

(c) Average speed is, in general, greater than the magnitude of average velocity.

The dimension of velocity and speed is $[LT^{-1}]$ and their SI unit is meters per second (m/s)

Example 2. In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find
(a) Average speed of Ram and Shyam?
(b) Average velocity of Ram and Shyam?

Sol. (a) Average speed of Ram $= \frac{100}{4}$ m/s = 25 m/s

Average speed of Shyam $= \frac{50\pi}{5}$ m/s = 10π m/s

(b) Average velocity of Ram $= \frac{100}{4}$ m/s = 25 m/s

Average velocity of Shyam $= \frac{100}{5}$ m/s = 20 m/s

Example 3. A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Sol. Let total distance travelled by the particle be 2s.

Time taken to travel first half $= \frac{s}{v_1}$

Time taken to travel next half $= \frac{s}{v_2}$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

Q.1 A particle covers $\frac{3}{4}$ of total distance with speed v_1 and next $\frac{1}{4}$ with v_2. Find the average speed of the particle?

Ans. $\frac{4v_1v_2}{v_1 + 3v_2}$

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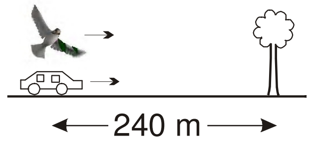
Q.2. A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the tree?



Ans. 360 m

2.6 Instantaneous Velocity (at an instant) :

The velocity at a particular instant of time is known as instantaneous velocity. The term *"velocity"* usually means instantaneous velocity.

$$V_{inst.} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

In other words, the instantaneous velocity at a given moment (say , t) is the limiting value of the average velocity as we let Δt approach zero. The limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the derivative of x with respect to t.

2.7 Average acceleration (in an interval):

The average acceleration for a finite time interval is defined as :

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

Average acceleration is a vector quantity whose direction is same as that of the change in velocity.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Since for a straight line motion the velocities are along a line, therefore

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

(where one has to substitute v_f and v_i with proper signs in one dimensional motion)

2.8 Instantaneous Acceleration (at an instant):

The instantaneous acceleration of a particle is its acceleration at a particular instant of time. It is defined as the derivative (rate of change) of velocity with respect to time. We usually mean instantaneous acceleration when we say *"acceleration"*. For straight motion we define instantaneous acceleration as :

$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right)$$

and in general $\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{v}}{\Delta t}\right)$

The dimension of acceleration is $[LT^{-2}]$ and its SI unit is m/s^2.

Example 4. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at t = 2 s?

Sol. Velocity; $v = \frac{dx}{dt} = 10t + 4$

At t = 2 s $v = 10(2) + 4 \Rightarrow v = 24$ m/s

Acceleration; $a = \frac{d^2x}{dt^2} = 10$

Acceleration is constant, so at t = 2 s

$a = z10$ m/s^2

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Q.3 The position of a particle moving on X-axis is given by

$$x = At^3 + Bt^2 + Ct + D.$$

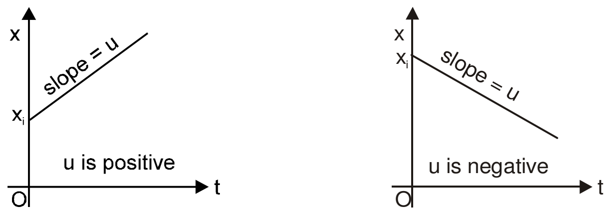
The numerical values of A, B, C, D are 1, 4, -2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t = 4 s, (c) the acceleration of the particle at t = 4s, (d) the average velocity during the interval t =0 to t = 4s, (e) the average acceleration during the interval t = 0 to t = 4 s.

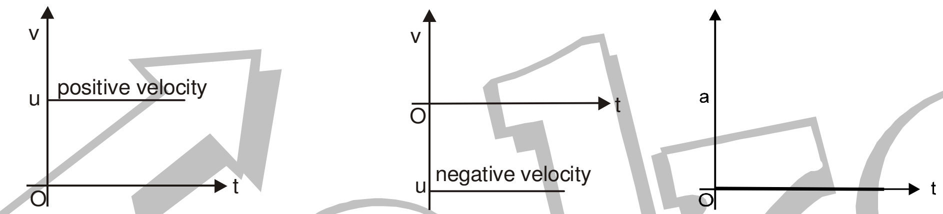
Ans. [(a) $[A] = [LT^{-3}]$, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$; (b) 78 m/s ; (c) 32 m/s^2 ; (d) 30 m/s ; (e) 20 m/s^2]

3. MOTION WITH UNIFORM VELOCITY

Consider a particle moving along x–axis with uniform velocity u starting from the point $x = x_i$ at t = 0.
Equations of x, v, a are : $x(t) = x_i + ut$; $v(t) = u$; $a(t) = 0$

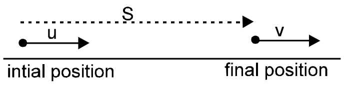
- x– t graph is a straight line of slope u through x_i.
- as velocity is constant, v – t graph is a horizontal line.
- a–t graph coincides with time axis because a = 0 at all time instants.





4. UNIFORMLY ACCELERATED MOTION

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.
For uniformly accelerated motion along a straight line (x–axis) during a time interval of t seconds, the following important results can be used.



(a) $v = u + at$
(b) $s = ut + 1/2\ at^2$
$s = vt - 1/2\ at^2$
$x_f = x_i + ut + 1/2\ at^2$
(c) $v^2 = u^2 + 2as$
(d) $s = 1/2\ (u + v)\ t$
(e) $s_n = u + a/2\ (2n - 1)$

u = initial velocity (at the beginning of interval)
a = acceleration
v = final velocity (at the end of interval)
s = displacement $(x_f - x_i)$
x_f = final coordinate (position)
x_i = initial coordinate (position)
s_n = displacement during the n^{th} sec

Example 5. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Sol. Let the total distance be 2x.
$\therefore$ distance upto midpoint = x
Let the velocity at the mid point be v
and acceleration be a.
From equations of motion

$v^2 = 10^2 + 2ax$ ____(1)
$30^2 = v^2 + 2ax$ ____(2)

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(2) - (1) gives

$v^2 - 30^2 = 10^2 - v^2 \quad \Rightarrow \quad v^2 = 500 \quad \Rightarrow \quad v = 10\sqrt{5}$ m/s

Example 6. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

Sol. Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \quad \text{\_\_\_\_ (1)}$$

During this interval the jeep travels a distance

$$s + d = vt \quad \text{\_\_\_\_ (2)}$$

By (1) and (2), $\frac{1}{2}at^2 + d = vt$

or, $t = \dfrac{v \pm \sqrt{v^2 - 2ad}}{a}$

The pickpocket will be caught if t is real and positive.

This will be possible if $v^2 \geq 2ad$ or, $v \geq \sqrt{2ad}$

Q.4 A car deccelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?
Ans. **[- 2 m/s²]**

Q. 5 A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?
Ans. **[50 m/s]**

5. GRAPHS IN UNIFORMLY ACCELERATED MOTION ($a \neq 0$)

• x is a quadratic polynomial in terms of t. Hence x – t graph is a parabola.

x-t graph

• v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.

v-t graph

• a–t graph is a horizontal line because a is constant.

a-t graph

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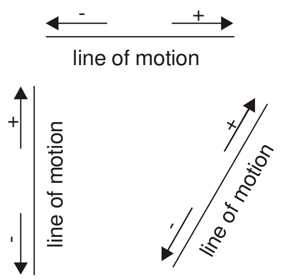
6. REACTION TIME

When a situation demands our immediate action. It takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

7. DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

- For example, if a particle is moving in a horizontal line (x–axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.



- For vertical or inclined motion, upward direction can be taken +ve and downward as –ve

- For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence acceleration for this type of motion will always be $a = -g$ i.e. $a = -9.8$ m/s^2 (–ve sign, because the force and acceleration are directed downwards, If we select upward direction as positive).

NOTE :

(a) If acceleration is in same direction as velocity, then speed of the particle increases.

(b) If acceleration is in opposite direction to the velocity then speed decreases i.e. the particle slows down. This situation is known as *retardation*.

Example 7. Mr. Sharma brake his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200m.
(a) How much time elapses during this interval?
(b) What is the acceleration?
(c) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Sol. (a) We select positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_i = 0$ when the braking begins. Then the initial velocity is $u_x = +25$ m/s at $t = 0$, and the final velocity and position are $v_x = +15$ m/s and $x = 200$ m at time t.
Since the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities.

$$\therefore \quad v_{av,x} = \frac{1}{2}(u_x + v_x) = \frac{1}{2}(15 + 25) = 20 \text{ m/s}.$$

The average velocity can also be expressed as $v_{av,x} = \frac{\Delta x}{\Delta t}$. With $\Delta x = 200$ m and $\Delta t = t - 0$, we can solve for t:

$$t = \frac{\Delta x}{v_{av,x}} = \frac{200}{20} = 10 \text{ s}.$$

(b) We can now find the acceleration using $v_x = u_x + a_x t$

$$a_x = \frac{v_x - u_x}{t} = \frac{15 - 25}{10} = -1 \text{ m/s}^2.$$

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(c) Now with known acceleration, we can find the total time for the car to go from velocity $u_x = 25$ m/s to $v_x = 0$. Solving for t, we find

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Successful People Replace the words like; "wish", "try" & "should" with "I Will". Ineffective People don't.

$$t = \frac{v_x - u_x}{a_x} = \frac{0 - 25}{-1} = 25 \text{ s.}$$

The total distance covered is

$$x = x_i + u_x t + \frac{1}{2} a_x t^2 \quad = 0 + (25)(25) + \frac{1}{2}(-1)(25)^2 = 625 - 312.5 = 312.5 \text{ m.}$$

Additional distance covered = 312.5 – 200 = 112.5 m.

Example 8. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower?

Sol. Let the total time of journey be n seconds.

Using; $$s_n = u + \frac{a}{2}(2n - 1)$$

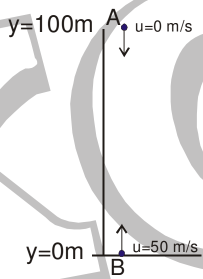
$$45 = 0 + \frac{10}{2}(2n - 1) \qquad n = 5 \text{ sec}$$

Height of tower;

$$h = \frac{1}{2} g t^2 \quad = \frac{1}{2} \times 10 \times 5^2 \quad = 125 \text{ m}$$

Example 9. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet?

Sol. Let the upward direction as positive.



Let the particles meet at a distance y from the ground.

For particle A,

$y_0 = +\,100$ m

$u = 0$ m/s

$a = -\,10$ m/s^2

$y = 100 + 0(t) - \frac{1}{2} \times 10 \times t^2$ $\qquad [y = y_0 + ut + \frac{1}{2}at^2]$

$= 100 - 5t^2$ ---- (1)

For particle B,

$y_0 = 0$ m $\Rightarrow$ $u = +\,50$ m/s $\Rightarrow$ $a = -\,10$ m/s^2

$y = 50(t) - \frac{1}{2} \times 10 \times t^2$

$= 50t - 5t^2$ ---- (2)

According to the problem;

$50t - 5t^2 = 100 - 5t^2$

$t = 2$ sec

Putting t = 2 sec in eqn. (1),

$y = 100 - 20$

$= 80$ m

Hence, the particles will meet at a height 80 m above the ground.

Q. 6 A particle is thrown vertically with velocity 20 m/s . Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.
Ans. [25m, 15m]

Q.7 A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 50 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s^2.
Ans. [68.5 m]

NOTE:- As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g.

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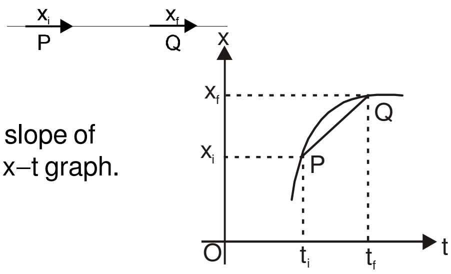
8. GRAPHICAL INTERPRETATION OF SOME QUANTITIES

8.1 Average Velocity

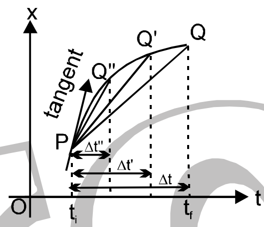
If a particle passes a point P (x_i) at time t = t_i and reaches Q (x_f) at a later time instant t = t_f, its average velocity in the interval PQ is $V_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$



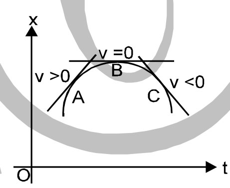
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x–t graph.

8.2 Instantaneous Velocity

Consider the motion of the particle between the two points P and Q on the x–t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ (Δt, $\Delta t'$, $\Delta t''$,......) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line (PQ, PQ′, PQ″......). As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P. As $\Delta t \to 0$, V_{av} $(=\Delta x/\Delta t) \to V_{inst.}$



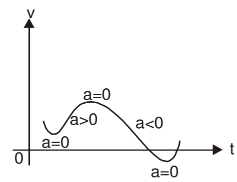
Geometrically, as $\Delta t \to 0$, chord PQ $\to$ tangent at P.



Hence the instantaneous velocity at P is the slope of the tangent at P in the x – t graph. When the slope of the x – t graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.

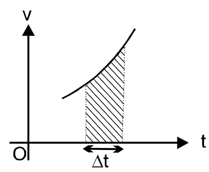
8.3 Instantaneous Acceleration

The derivative of velocity with respect to time is the slope of the tangent in velocity time (v–t) graph.



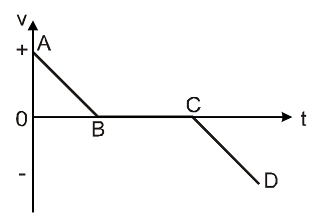
8.4 Displanncement from v - t graph

Displacement = Δx = area under v–t graph.
Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that $\Delta v = a\,\Delta t$ leads to the conclusion that **area under a – t graph gives the change in velocity Δv during that interval.**

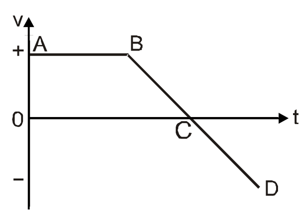


Example 10. Describe the motion shown by the following velocity-time graphs.

(a)



(b)



Sol. (a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down

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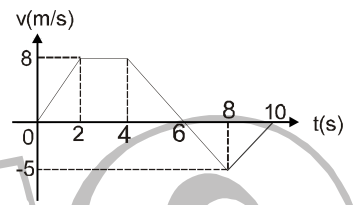
as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Example 11. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



Sol. Distance travelled = Area under v-t graph (taking all areas as +ve.)

$\therefore$ Distance travelled = Area of trapezium + Area of triangle

$$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5$$

$= 32 + 10 \quad = 42$ m

Displacement = Area under v-t graph (taking areas below time axis as -ive.)

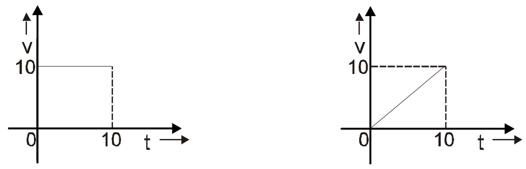
$\therefore$ Displacement = Area of trapezium – Area of triangle

$$= \frac{1}{2}(2+6)\times 8 - \frac{1}{2}\times 4\times 5$$

$= 32 - 10 \quad = 22$ m

Hence, distance travelled = 42 m and displacement = 22 m.

Q.8 For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



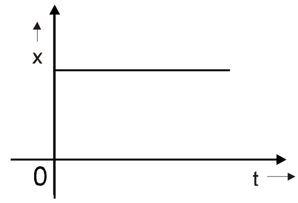
(a) (b)

Ans. [(a) 100m; (b) 50m]

9. Interpretation of some more Graphs

9.1 Position vs Time graph

9.1.1 Zero Velocity



As position of particle is fix at all the time, so the body is at rest.

Slope; $\frac{dx}{dt} = \tan\theta = \tan 0^{\circ} = 0$

Velocity of particle is zero

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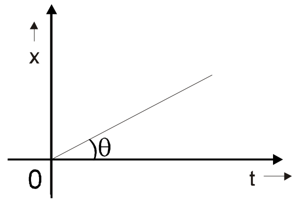
9.1.2 Uniform Velocity

Here tanθ is constant $\tan\theta = \frac{dx}{dt}$

$\therefore$ $\frac{dx}{dt}$ is constant.

$\therefore$ velocity of particle is constant.



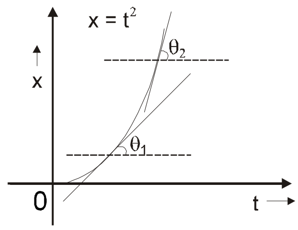
9.1.3 Non uniform velocity (increasing with time)

In this case;

As time is increasing, θ is also increasing.

$\therefore$ $\frac{dx}{dt}$ = tanθ is also increasing

Hence, velocity of particle is increasing.



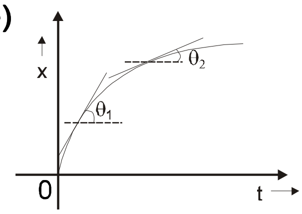
9.1.4 Non uniform velocity (decreasing with time)

In this case;

As time increases, θ decreases.

$\therefore$ $\frac{dx}{dt}$ = tanθ also decreases.

Hence, velocity of particle is decreasing.



9.2 Velocity vs time graph

9.2.1 Zero acceleration

Velocity is constant.

tanθ = 0

$\therefore$ $\frac{dv}{dt} = 0$

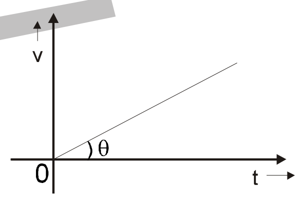
Hence, acceleration is zero.

9.2.2 Uniform acceleration

tanθ is constant.

$\therefore$ $\frac{dv}{dt}$ = constant

Hence, it shows constant acceleration.



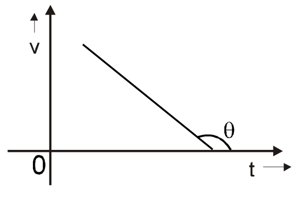
9.2.3 Uniform retardation

Since θ > 90º

$\therefore$ tanθ is constant and negative.

$\therefore$ $\frac{dv}{dt}$ = negative constant

Hence, it shows constant retardation.



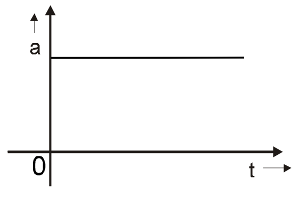
9.3 Acceleration vs time graph

9.3.1 Constant acceleration

tanθ = 0

$\therefore$ $\frac{da}{dt} = 0$

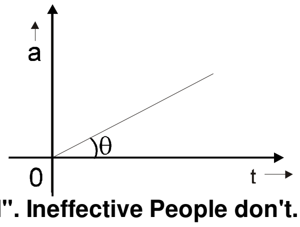
Hence, acceleration is constant.



9.3.2 Uniformly increasing acceleration

θ is constant.

$0º < \theta < 90º \Rightarrow \tan\theta > 0$



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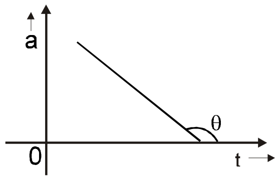
$\therefore \quad \frac{da}{dt} = \tan\theta = \text{constant} > 0$

Hence, acceleration is uniformly increasing with time.

9.3.3 Uniformly decreasing acceleration



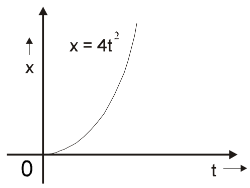
Since $\theta > 90^\circ$

$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \quad \frac{da}{dt}$ = negative constant

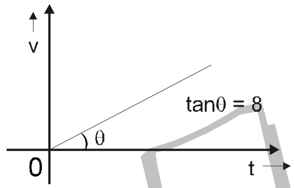
Hence, acceleration is uniformly decreasing with time

Example 12. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



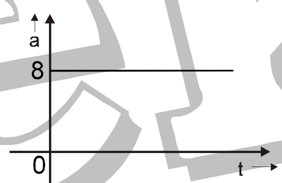
Sol: $x = 4t^2 \quad \Rightarrow \quad v = \frac{dx}{dt} = 8t$

Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.

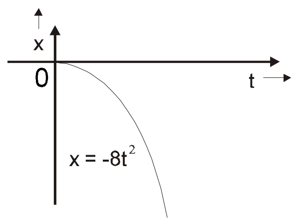


$a = \frac{dv}{dt} = 8$

Hence, acceleration is constant throughout and is equal to 8.



Problem 10. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



11. MOTION WITH NON-UNIFORM ACCELERATION (use of definite integrals)

$$\Delta x = \int_{t_i}^{t_f} v(t)\,dt \qquad \text{(displacement in time interval } t = t_i \text{ to } t_f)$$

The expression on the right hand side is called the *definite integral* of v(t) between $t = t_i$ and $t = t_f$. . Similarly change in velocity

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)\,dt$$

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Successful People Replace the words like; "wish", "try" & "should" with "I Will". Ineffective People don't.

Table 2 : Some quantities defined as derivatives and integrals.

$v(t) = \frac{dx}{dt}$	v = slope of x–t graph
$a(t) = \frac{dv}{dt}$	a = slope of v–t graphs
$F(t) = \frac{dp}{dt}$	F = slope of p–t graph (p = linear momentum)
$\Delta x = \int dx = \int_{t_i}^{t_f} v(t)\,dt$	Δx = area under v–t graph
$\Delta v = \int dv = \int_{t_i}^{t_f} a(t)\,dt$	Δv = area under a–t graph
$\Delta p = \int dp = \int_{t_i}^{t_f} F(t)\,dt$	Δp = area under F–t graph
$W = \int dW = \int_{x_i}^{x_f} F(x)\,dx$	W = area under F–x graph

12. SOLVING PROBLEMS WHICH INVOLVES NONUNIFORM ACCELERATION

12.1 Acceleration depending on velocity v or time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t, $\int_{v_0}^{v} dv = \int_0^t a(t)\,dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$. On integrating, we get a relation between v and t, and then using $\int_{x_0}^{x} dx = \int_0^t v(t)\,dt$, x and t can also be related.

12.2 Acceleration depending on velocity v or position x

$$a = \frac{dv}{dt} \Rightarrow a = \frac{dv}{dx}\frac{dx}{dt} \Rightarrow a = \frac{dx}{dt}\frac{dv}{dx} \Rightarrow a = v\frac{dv}{dx}$$

This is another important expression for acceleration.

If a is in terms of x, $\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$.

If a is in terms of v, $\int_{v_0}^{v} \frac{v\,dv}{a(v)} = \int_{x_0}^{x} dx$

On integrating, we get a relation between x and v. Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_0^t dt$, we can relate x and t.

Example 13. An object starts from rest at t = 0 and accelerates at a rate given by a = 6t. What is
(a) its velocity and
(b) its displacement at any time t?

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Sol. As acceleration is given as a function of time,

$\therefore \quad \int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a(t)dt$

Here $t_0 = 0$ and $v(t_0) = 0$

$\therefore \quad v(t) = \int_0^t 6tdt \quad = 6\left(\frac{t^2}{2}\right)\Big|_0^t \quad = 6\left(\frac{t^2}{2} - 0\right) \quad = 3t^2$

So, $v(t) = 3t^2$

As $\quad \Delta x = \int_{t_0}^{t} v(t)dt$

$\therefore \quad \Delta x = \int_0^t 3t^2 dt \quad = 3\left(\frac{t^3}{3}\right)\Big|_0^t \quad = 3\left(\frac{t^3}{3} - 0\right) \quad = t^3$

Hence, velocity $v(t) = 3t^2$ and displacement $\Delta x = t^3$

Q.9. For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at $x = 0$, find speed as a function of x.

Ans. [$v = v_0 e^{2x}$]

Q. 10. For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At $t = 0$, particle is at origin. Find the position as a function of time?

Ans. [$x = \frac{2}{3}t^3 - \cos(t) + 1$]

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SUMMARY

Rectilinear Motion: Rectilinear motion is motion, along a straight line or in one dimension.

Displacement : The vector joining the initial position of the particle to its final position during an interval of time.

Distance : The length of the actual path travelled by a particle during a given time interval

Average Velocity $= \dfrac{\text{displacement}}{\text{time interval}} = \dfrac{x_f - x_i}{t_f - t_i}$

Average Speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$

Instantaneous Velocity : $V_{inst.} = \lim_{\Delta t \to 0} \left(\dfrac{\Delta x}{\Delta t}\right) = \dfrac{dx}{dt}$

Average Acceleration

$$= \frac{\text{change in velocity}}{\text{time interval}} = \frac{v_f - v_i}{t_f - t_i}$$

Instantaneous Acceleration :

$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right)$$

Equations of Motion

(a) $v = u + at$

(b) $s = ut + 1/2\, at^2$

$s = vt - 1/2\, at^2$

$x_f = x_i + ut + 1/2\, at^2$

(c) $v^2 = u^2 + 2as$

(d) $s = 1/2\,(u + v)\, t$

(e) $s_n = u + a/2\,(2n - 1)$

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Maxima and Minima

Conditions for maxima are:–

$$\frac{dy}{dx} = 0 \quad (b)\ \frac{d^2y}{dx^2} < 0$$

Conditions for minima are:–

$$\frac{dy}{dx} = 0 \quad (b)\ \frac{d^2y}{dx^2} > 0$$

Motion with Non-Uniform Acceleration

$$\Delta x = \int_{t_i}^{t_f} v(t)\,dt$$

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)\,dt$$

Solving Problems which Involves Nonuniform Acceleration

If a is in terms of t, $\displaystyle\int_{v_0}^{v} dv = \int_0^t a(t)\,dt$

If a is in terms of v, $\displaystyle\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$

If a is in terms of x, $\displaystyle\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$.

If a is in terms of v, $\displaystyle\int_{v_0}^{v} \frac{v\,dv}{a(v)} = \int_{x_0}^{x} dx$

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